

On Observables in a Dark Matter-Clustering Quintessence System

Matteo Fasiello^a and Zvonimir Vlah^{a,b}

^a*Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94306 and*

^b*Kavli Institute for Particle Astrophysics and Cosmology,
Stanford University and SLAC, Menlo Park, CA 94025*

We consider a system where dark matter (DM) dynamics is enriched by the presence of clustering quintessence in the approximation where the system is effectively reduced to one degree of freedom. We focus on the behaviour of the one-loop total density power spectrum in the IR limit and on the so-called *consistency conditions* (*ccs*). We find that the power spectrum shows an enhancement in the IR with respect to the pure dark matter case and suggest a parallel with the behaviour of the non-equal time pure (DM) correlator. We then analyze *ccs* for a more general setup and recover the result that, for $c_s = w$, *ccs* are conserved outside the horizon but generically not inside. We extend these results. In these and similar scenarios the presence of additional dynamics (e.g. dark energy, modified gravity) implies that one may not “gauge away” the squeezed contribution of observables such as the dark matter bispectrum. We comment on how these effects may propagate all the way to biased tracers observables.

I. INTRODUCTION

A detailed understanding of the nonlinear formation of structure in the Universe is of paramount importance for cosmology. These scales carry crucial information both on early (e.g. non-Gaussianities [1]) and late-time (e.g. current cosmic acceleration [2]) physics. Several approaches have been developed to tackle the dynamics in these regimes, from N-body simulations to perturbative frameworks [3]–[13]. In what follows we adopt the latter treatment to investigate the effects of adding a clustering quintessence component to dark matter in the fluid description. Such setup enjoys some simplifications in the treatment that do not necessarily generalize. Nevertheless, it should be considered as an illustrative proxy for systems exhibiting a richer dynamics.

II. TOTAL POWER SPECTRUM

Additional degrees of freedom (e.g. those of dynamical dark energy) can be added to the fluid description of clustering dark matter, thereby generating a system of gravitationally coupled equations. In the next section, we will describe some of the properties of such a system. In this one we study the simplified, reduced, dynamics of a clustering quintessence model in the vanishing sound speed approximation first analyzed in [14]. The continuity and Euler equation read

$$\begin{aligned}\partial_\tau \delta_T + \partial_i [(C + \delta_T) v^i] &= 0, \\ \partial_\tau v^i + \mathcal{H} v^i + v^j \partial_j v^i &= -\nabla^i \Phi,\end{aligned}\quad (1)$$

where $C = 1 + (1 + w) \frac{\Omega_Q}{\Omega_m}$ and the system is closed with the Poisson equations $\nabla^2 \Phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \delta_T$. The dynamics can be solved perturbatively. The results for the three level bispectrum are given in [14] whilst the one loop power spectrum, as well as all-order integral solutions for the fields, were found in [15]. The kernels for the

total density fluctuations δ_T up to the third order are

$$\begin{aligned}F_2 &= -\frac{1}{2} \left(1 - \epsilon^{(1)} - \frac{3}{2} \nu_2 \right) \alpha_s + \frac{3}{2} \left(1 - \epsilon^{(1)} - \frac{1}{2} \nu_2 \right) \beta, \\ F_3 &= (1 - \epsilon^{(2)}) \mathcal{F}_3^\epsilon + \nu_3 \mathcal{F}_3^{\nu_3} + (1 - \epsilon^{(1)}) \nu_2 \mathcal{F}_3^{\nu_2} \\ &\quad + \lambda_1 \mathcal{F}_3^{\lambda_1} + \lambda_2 \mathcal{F}_3^{\lambda_2},\end{aligned}\quad (2)$$

where for simplicity we have suppressed momentum-vector dependence as well as time dependence in $\epsilon, \nu_3, \nu_2, \lambda_2$ and λ_1 (for explicit definitions we refer the reader to [14, 15]). The reduced kernels α and β and \mathcal{F}_3^ϵ etc are time-independent and are only function of momenta [15]. We reproduce here the explicit form of the quantities $\epsilon^{(n)}$ since they will be of particular importance in what follows. These read: $\epsilon^{(1)}(\eta) = 1 - e^{-\eta} \int_{-\infty}^{\eta} d\tilde{\eta} [e^{\tilde{\eta}} / C(\tilde{\eta})]$ and, similarly, $\epsilon^{(2)} = 2 \int_{-\infty}^{\eta} d\tilde{\eta} e^{2(\tilde{\eta}-\eta)} (1 - (1 - \epsilon)/C(\tilde{\eta}))$. Note that, by construction, both vanish in the simplifying case where $C = 1$, as the system in Eq. (1) reduces to the pure dark matter case. We consider the one-loop power spectrum for the total density:

$$P_{1-\text{loop}}(k, a) = P_L(k, a) + P_{22}(k, a) + 2P_{13}(k, a) + P_{\text{c.t.}}(k, a),\quad (3)$$

where each of the above contributions is defined as

$$\begin{aligned}P_{L,k}(a) &= D_+^2(a) P_{\mathbf{k}}^{\text{in}}, \\ P_{22,k}(a) &= 2D_+^4(a) \int_{\mathbf{q}} [F_2(\mathbf{k} - \mathbf{q}, \mathbf{q}, a)]^2 P_{|\mathbf{k}-\mathbf{q}|}^{\text{in}} P_{\mathbf{q}}^{\text{in}}, \\ P_{13,k}(a) &= 3D_+^4(a) P_{\mathbf{k}}^{\text{in}} \int_{\mathbf{q}} F_3(\mathbf{k}, -\mathbf{q}, \mathbf{q}, a) P_{\mathbf{q}}^{\text{in}}.\end{aligned}\quad (4)$$

Here $P_{\text{c.t.}}(k, a)$ stands for the one-loop *counterterm*, encoding short-scale dynamics. It is given in the *EFTofLSS* [12] simply as $\propto k^2/k_{\text{NL}}^2 P_L$, and multiplies a to-be-determined (via e.g. comparison with N-body) numerical coefficient. P_k^{in} is the time-independent initial power spectrum obtained from Boltzmann algorithms such as [16, 17].

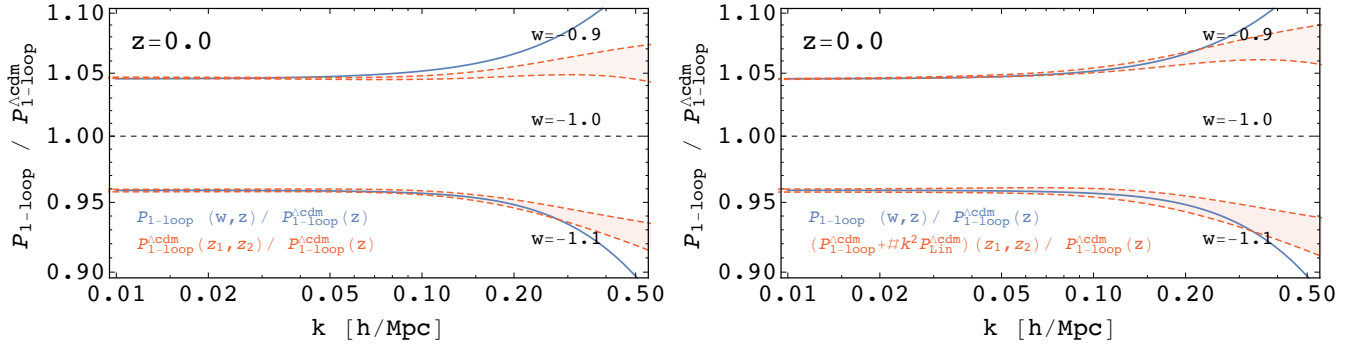


FIG. 1: The ratio of the total density power spectrum for $w = -1.1$ or -0.9 and the ΛCDM ($w = -1.0$) power spectrum is shown in blue solid lines. *Left:* Red dashed lines and red band represent the approximation obtained from the non-equal time ΛCDM power spectrum choosing z_1 and z_2 different from the nominal z . *Right:* In addition to the non-equal time ΛCDM power spectrum, red dashed lines and red band contain an additional small contributions $\sim k^2 P_{\text{lin}}$. This corresponds to the small change of value of the EFT parameter, which can further extend the k -range validity of this approximation.

In ΛCDM , it is well-known that a cancellation between the leading IR contributions occurs [18–22]. Our system describes a richer dynamics, as exemplified by the presence of the time-dependent $C(\tau)$ in Eq.(1). It is then interesting to study the same limit. Indeed, for the one-loop leading IR contributions we find

$$\left. \frac{p_{k,q}^{1\text{-loop}}(a)}{P_{L,k}(a)P_{L,q}(a)} \right|_{k \gg q} \sim \left(\left(1 - \epsilon^{(1)} \right)^2 - 1 + \epsilon^{(2)} \right) \frac{2}{3} \frac{k^2}{q^2}, \quad (5)$$

where $p_{k,q}^{1\text{-loop}}$ is the integrand of the one-loop power spectrum: $P_k^{1\text{-loop}} = \int_{\mathbf{q}} p_{k,q}^{1\text{-loop}}$. For $C(\tau) = 1$, i.e. in the ΛCDM limit, we recover the expected cancellation. It is interesting to note that also a generic constant C will result in a cancellation, indicating that only non-trivial dynamics on top of that of DM leads to an enhanced IR power spectrum.

The behaviour that a clustering quintessence component, switched on at an appropriately small redshifts, enforces on the total power spectrum, is reminiscent of the *non-equal time* ΛCDM dark matter power spectrum. Indeed, in the same IR limit, we obtain for the latter

$$\left. \frac{p_{1\text{-loop},k,q}(a_1, a_2)}{P_{L,k}(a_1)P_{L,q}(a_2)} \right|_{k \gg q} \sim - \frac{(D(a_1) - D(a_2))^2}{D(a_1)D(a_2)} \frac{1}{3} \frac{k^2}{q^2}, \quad (6)$$

where D is the linear growth function. As we shall clarify in detail in *Section III*, the one between these two systems in the IR is not an exact equivalence. The parallel between the infrared power spectrum of the DM+Quintessence duo and that of non-equal time DM fields stems from the fact that a non-trivial time-dependence in $C(\tau)$ acts as an additional “clock” in the DM-only dynamics. As such, it mimics non-equal time-dependent observables. In Fig. 1 we continue with the parallel and show the total (DM+CQ) one loop power spectrum results for different w , at redshift $z = 0$, as

a function of momentum. In the same plots we show the non-equal time power spectrum for the ΛCDM case. One can see that the two power spectra are very similar ($\ll 1\%$ difference) on scales where the one-loop results are expected to be valid, $k \lesssim 0.15 \text{ Mpc/h}$ [12]. On the right panel we add a typical counterterm contribution. This shows that a small change in the counterterm parameter values assures that an unequal-time power spectrum can mimic our DM+CQ 10% deviations from (equal time) ΛCDM up to and beyond $k \lesssim 0.2 \text{ Mpc/h}$.

The IR (or, in general, *squeezed*) behaviour of cosmological correlators is best understood by studying how it emerges from the symmetries (or lack thereof) of a physical system. We expand on this topic below.

III. CCS IN CLUSTERING QUINTESSENCE

In this section we follow the notation of [23],[24]. Consistency conditions (hereafter *ccs*) stem from a residual gauge symmetry of the action or the equations of motion (eom) of a physical system. Although certain gauges, such as unitary gauge, are known to completely fix diffeomorphism (diff) invariance, the fact that such fixing is complete is strictly true only for diffs that vanish at spatial infinity. Indeed, the residual gauge symmetry *ccs* rely upon is that of diffs that do not vanish at infinity [25].

One may derive non-trivial *ccs* when the soft mode characterizing any squeezed limit transforms non-linearly under the residual diff. In the context of Large Scale Structure, using the fluid treatment for pure dark matter dynamics, one may show that the system eom possesses a time dependent symmetry under which the velocity potential π and the Newtonian potential Φ transform non-linearly [20, 21],[24]. It follows that the effect of a long mode π_L on n short modes corresponds to the action of a residual gauge-symmetry on the observable made up by the corresponding n -point function and can, as such, be gauged

away. This often translates into a suppressed signal for the squeezed $n+1$ -correlator.

The mere requirement that the eom are invariant under a residual diff is not enough to guarantee that the *ccs* (at least in their standard formulation), are in place. Following e.g. [24] one may list three main necessary conditions:

- Symmetry of the action (eom) under the residual diff;
- Single-clockness: the transformation of an array of n hard modes is mapped to the presence of one soft mode;
- Adiabaticity: the eom of the gauge parameter describing the residual diff mimics that of a long physical mode.

Our reduced system will put these requirements to the test. To clarify the picture and set the stage for generalizations, we report the full system in Eq.(7) below, in the approximation already in use in [14] (see also [26]) and in the Newtonian limit:

$$\begin{aligned}
\frac{\partial \delta_m}{\partial \tau} + \partial_i [(1 + \delta_m) v_m^i] &= 0, \\
\frac{\partial \delta_Q}{\partial \tau} - 3(w - c_s^2) \mathcal{H} \delta_Q + \partial_i \{ [(1 + \omega) + (1 + c_s^2) \delta_Q] v_Q^i \} &= 0, \\
\frac{\partial v_m^i}{\partial \tau} + \mathcal{H} v_m^i + v_m^j \partial_j v_m^i &= -\nabla^i \Phi, \\
\nabla^2 \Phi &= \frac{3}{2} \mathcal{H}^2 \Omega_m \left(\delta_m + \frac{\Omega_q}{\Omega_m} \delta_Q \right) \equiv \frac{3}{2} \mathcal{H}^2 \Omega_m \delta_T, \\
v_Q^{i'} + \mathcal{H}(1 - 3w) v_Q^i + v_Q^j \partial_j v_Q^i &= -\partial_i \Phi - \frac{c_s^2 \partial_i \delta_Q}{1 + w}.
\end{aligned} \tag{7}$$

Linearly, the system of eom [36] is invariant under time-dependent translations:

$$\begin{aligned}
\tau \rightarrow \tilde{\tau} = \tau; \quad x^i \rightarrow \tilde{x}^i = x^i + n^i(\tau); \quad v_{m,Q}^i \rightarrow v_{m,Q}^i + n^{i'} \\
\delta_{m,Q} \rightarrow \tilde{\delta}_{m,Q} = \delta_{m,Q}; \quad \Phi \rightarrow \tilde{\Phi} = \Phi - x^i (\mathcal{H} n^{i'} + n^{i''}).
\end{aligned} \tag{8}$$

This is precisely the kind of symmetry that can generate *ccs*. One needs to be particularly watchful when multiple degrees of freedom (dof) make up the dynamics. In this case it is possible that the residual diff does not generate a relation between just one specific three-point function and the variation of the corresponding two-point correlator. What happens is that a linear combination of different squeezed three-point functions may correspond to the 2-point function variation. This makes the content of *ccs* somewhat weaker as one may no longer deduce information on one single “squeezed” observable. One key ingredient for *ccs* to be in place is the existence of a field that transforms (also) non linearly under the residual diff “ s ”. We will be schematic so as to avoid clutter; in coordinate space:

$$\delta_s \varphi = [Q_s, \varphi] \sim \underbrace{(\dots) \varphi}_{\text{linear}} + \underbrace{(\dots)}_{\text{non-linear}}, \tag{9}$$

where above the dots preceding φ stand typically for a differential operator and the non-linear piece is not proportional to φ [37]. Let us now apply this transformation

to the 2-point function made up by two hard modes:

$$\langle [Q_s, \varphi \varphi] \rangle \sim \langle \varphi \varphi \rangle + \underbrace{\dots}_{\text{non-linear} + \text{mixed}}, \tag{10}$$

where the unspecified non-linear terms on the RHS stand for pieces corresponding to diagrams which are not *connected* and we omit in what follows. There is another way to express the action of the symmetry via the charge Q_s ; it relies on introducing a complete set of mutually orthogonal “states” [23] (see also [28]):

$$\langle [Q_s, \varphi \varphi] \rangle \sim 2i \text{Im} \left[\langle Q_s \sum_n (|n\rangle \langle n|) \varphi \varphi \rangle \right]. \tag{11}$$

Let us limit here the number of such states to two: $|\varphi\rangle$ and $|\sigma\rangle$, which we assume orthogonalized. Combining Eq. (10) and (11), it follows

$$\frac{\partial}{\partial} \langle \varphi \varphi \rangle \sim 2i \text{Im} \left[\langle Q_s |\varphi\rangle \langle \varphi | \varphi \varphi \rangle + \langle Q_s |\sigma\rangle \langle \sigma | \varphi \varphi \rangle \right]. \tag{12}$$

Now, if φ and σ transform only linearly under the symmetry, by virtue of the averaging process one has

$$\langle Q_s |\varphi\rangle \sim \langle \varphi \rangle = 0 = \frac{\partial}{\partial} \langle \varphi, \varphi \rangle, \tag{13}$$

and no non-trivial *ccs*. If instead only e.g. φ has a non-linear transformation component

$$\langle Q_s |\varphi\rangle = c_{\text{number}}(q \rightarrow 0) \Rightarrow \frac{\partial}{\partial} \langle \varphi \varphi \rangle \sim \langle \varphi_{q \rightarrow 0} \varphi \varphi \rangle, \tag{14}$$

that is, standard *ccs* hold. Finally, if also σ has a non-linear component the result would be a less standard:

$$\frac{\partial}{\partial} \langle \varphi \varphi \rangle \sim \langle \varphi_{q \rightarrow 0} \varphi \varphi \rangle + c_r \langle \sigma_{q \rightarrow 0} \varphi \varphi \rangle, \tag{15}$$

where c_r is for a relative coefficient, unimportant for the present discussion. If both terms in Eq.(15) are non-zero one concludes that the single-clock requirement is no longer met and thus the squeezed contribution of quantities such as $\langle \varphi_{q \rightarrow 0} \varphi_k \varphi_{q-k} \rangle$ may not be gauged away. This carries important observational consequences which are well known e.g. in the context of multi-field inflationary models where *ccs* breaking contains information on the mass and the spin of the involved particles [29, 30].

Let us now apply this line of reasoning to the system in Eq. (7). From Eq. (8) it is clear that there are several modes that transform non-linearly under the symmetry: Φ, π_m, π_Q , the last two being the velocity potentials ($v_i = \partial_i \pi$) of v_m, v_Q . Furthermore the two velocities (potentials) have generically independent solutions (in the above terminology, they can be orthogonalized). In the most general setup the action of a soft mode is then of the form in Eq. (15) and does therefore *break* standard *ccs*. We stress that such a setup is not at all limited to a quintessence component but applies to e.g. general dark energy and modified gravity dynamics, with intriguing exceptions such as Galileons [31].

We now ask what happens in a reduced system such as that of Eq.(1). The latter can be obtained by imposing on the full system that $c_s \rightarrow 0$ and $v_m = v_Q$. The assumption of a common velocity for all species is here akin to having, loosely speaking, one and a half degrees of freedom. In the constant w case, the eom for the total density $\delta_T = \delta_m + \frac{\Omega_Q}{\Omega_m} \delta_Q$ reduces to Eq.(1). It enjoys the same invariance under Eq.(8) but there is only one velocity potential and its action as a soft mode is not independent from that of the Newtonian potential as one may readily verify from the linear eom. The two act in fact as alternative soft “pions” [24]. What is less evident is whether the adiabaticity condition holds: the soft physical mode and the gauge parameter sharing the same equation of motion. This may not happen at all or, in particular, it may occur only outside the horizon. A quick route to the answer is the following.

- (1) The adiabaticity condition is known to be satisfied in the pure dark matter case in that the velocity potential equation and gauge parameter eom are the same;
- (2) The gauge parameter eom does not depend on w [24];
- (3) Although the velocity eom in Eq.(1) is formally the same as the DM Euler equation in Eq. (7), the resulting solution will not be because the respectively coupled continuity equations will be different as soon as $C(\eta)$ has a non-trivial time dependence in Eq.(1). It follows that the adiabaticity condition is not met and, as seen in *Section II*, ccs are broken.

Let us now provide some more details. We make explicit below the eom for quintessence and relax the Newtonian limit assumption. Anticipating that we are going to use the modes in the soft limit, we limit the analysis to the linear eoms. We first integrate over time and solve for δ_Q the quintessence density contrast eom

$$\delta'_Q - 3(w - c_s^2)\mathcal{H}\delta_Q + \partial_i[(1 + \omega)v^i] = 3\Psi'(1 + w). \quad (16)$$

We then write Euler equation for the velocity potential:

$$\pi' + \mathcal{H}(1 - 3w)\pi = -\Phi - \frac{c_s^2 \delta_Q}{1 + w}. \quad (17)$$

Recognizing that $\Phi = \Psi$ corresponds to the absence of anisotropic stress, an assumption we make here, one may solve Eq. (16) and plug into Eq. (17) to derive an expression for Φ . The latter is then employed in the $(0, i)$ component of the perturbed Einstein equations (for the scalar sector):

$$-(\Psi' + \mathcal{H}\Psi) = 4\pi G a^2 \sum_{\alpha} (\bar{\rho}_{\alpha} + \bar{p}_{\alpha}) \pi_{\alpha}, \quad (18)$$

where a conservation law for each dynamical species has been assumed. In particular, the common velocity requirement allows us to factor $\pi_{\alpha} = \pi$ out of the equation above. The resulting equation is:

$$\begin{aligned} & (\pi' + 2\mathcal{H}\pi - B)' - 3w\mathcal{H}(\pi' + \mathcal{H}\pi + \frac{\mathcal{H}'}{\mathcal{H}}\pi) \\ & - 3c_s^2 \mathcal{H}(\mathcal{H}\pi - \frac{\mathcal{H}'}{\mathcal{H}}\pi - B) - c_s^2(g' + \mathcal{H}g) \\ & - 3c_s^2 \frac{(c_s^2 - w)}{1 + w} \left[\mathcal{H}\delta_Q + \mathcal{H} \int d\tau \mathcal{H}\delta_Q \right] = 0, \end{aligned} \quad (19)$$

where B is a constant and we have added and subtracted a term proportional to $(\mathcal{H}^2 - \mathcal{H}')\pi$, which is useful in light of the Friedman equation

$$\mathcal{H}^2 - \mathcal{H}' = 4\pi G a^2 \sum_{\alpha} (\bar{\rho} + \bar{p}). \quad (20)$$

The function g is defined as $g = \int d\tau \nabla^2 \pi$ and is therefore important only inside the horizon. Let us now analyze Eq. (19) in some detail. For $c_s = 0 = w$ one has:

$$\pi' + 2\mathcal{H}\pi - C = 0. \quad (21)$$

This is precisely the same equation of the gauge parameter $n^{i'}$ in Newtonian gauge [24]. Indeed, for n^i one has:

$$n^{i''} + 2\mathcal{H}n^{i'} + 2b^i = 0, \quad (22)$$

with b^i constant, recovering the well-known result that all three criteria are met in Λ CDM and ccs are active. In the special case $w = c_s$ we see that the third line in Eq. (19) vanishes and outside the horizon π can satisfy Eq. (21) for any w : adiabaticity is satisfied and we reproduce the findings of [24]. We stop at this stage to note that in the full system of Eq. (7) the w -proportional term in the Euler quintessence equation, as well as the $c_s^2 \partial_i \delta v^i$ in the second continuity equation, prevent these relations from being fully invariant under Eq.(8) [38]. And yet, we know that for $c_s^2 = w$ ccs are active outside the horizon. This has to do with the common velocity requirement. In the Newtonian limit it amounts to, subtracting DM and quintessence continuity equations, requiring that

$$3w\mathcal{H}v^i \simeq \frac{c_s^2 \partial^i \delta_Q}{1 + w}. \quad (23)$$

Outside the horizon the RHS vanishes and, because of the constraint, so does the LHS. It follows that, at very large distances and for $w = c_s^2$, the common velocity constraint dynamically enforces the symmetry. The adiabaticity and initial conditions requirement are also satisfied and so are, in turn, ccs . Finally, upon inspecting Eq. (19) we see that for $c_s = 0$ but generic w it is no longer possible to satisfy Eq.(21) even outside the horizon, thereby concluding that for the reduced system in Eq. (1) ccs are indeed broken [39].

IV. CLUSTERING OF BIASED TRACERS

One may ask how the effects of additional dofs propagate all the way to biased tracers observables. Just as in Λ CDM, here too the effect of short distance physics on long wavelength dynamics is encoded in an effective stress-energy tensor. There is however also an effective force: it accounts for the momentum exchange (at short distances) between dark matter and additional species, mediated by gravity. One can employ the bias models developed in [32–35] to derive the results for two or more species. Besides the gravitational field, the density and

velocity gradients, as well as the relative velocity of the two species, appear now in

$$\begin{aligned} \delta_h(\vec{x}, t) \simeq \int^t H(t') \left[c_{\delta_T}(t') \frac{\delta_T(\vec{x}_{\text{fl}}, t')}{H(t')^2} + c_{\delta_{\text{d.e.}}}(t') \delta_{\text{d.e.}}(\vec{x}_{\text{fl}}) \right. \\ + c_{\partial v_c}(t') \frac{\partial_i v_c^i(\vec{x}_{\text{fl}}, t')}{H(t')} + c_{\partial v_{\text{d.e.}}}(t') \frac{\partial_i v_{\text{d.e.}}^i(\vec{x}_{\text{fl}}, t')}{H(t')} \\ + c_{\epsilon_c}(t') \epsilon_c(\vec{x}_{\text{fl}}, t') + c_{\epsilon_{\text{d.e.}}}(t') \epsilon_{\text{d.e.}}(\vec{x}_{\text{fl}}, t') \\ \left. + c_{\partial^2 \delta_T}(t') \frac{\partial_{x_{\text{fl}}}^2 \delta_T(\vec{x}_{\text{fl}}, t')}{k_{\text{M}}^2 H(t')^2} + \dots \right]. \quad (24) \end{aligned}$$

Note also that, in principle, the definition of the “flow” variable, \vec{x}_{fl} , accounting for the halo formation, can be different for different species [35]. The above equation resembles the DM+baryons result, although there exist one important difference. The deviation from the EdS-like approximation (i.e. assuming that kernels in Eq. (2) are time-independent) introduces new operators in the bias expansion above. Indeed, upon using Eq. (24), one can show that the time dependence of the kernels in Eq. (2), encoded in terms such as $\epsilon^{(1)}$, $\epsilon^{(2)}$, ν_3 , introduces (after formally performing the time integration) an indepen-

dent bias coefficient (see [35]). At second order in the field $\delta_h^{(2)}$, this is not expected to yield any new independent bias operators due to degeneracies in the operators momentum dependence. For $\delta_h^{(3)}$, on the other hand, we expect one new independent operator to arise due to the effects of time evolution. This same reasoning holds already in Λ CDM cosmology (if one does *not* assume the EdS-like approximation) but there the effects are known to be small. Crucially, the presence of additional dofs, such as dark energy, will magnify them.

Conclusions

We have shown how the additional dynamics typical of e.g. beyond- Λ CDM models may affect cosmological observables. It is crucial to be aware of such imprints and possible degeneracies these might have with e.g. primordial sources to make the best possible use of the upcoming data from astronomical surveys.

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 - [35] R. Angulo et al, JCAP **1509**, no. 09, 029 (2015)
 - [36] Except for the w -proportional term on the LHS of the last relation and the $c_s^2 \partial_i \delta_Q v_Q^i$ (whose transformation has a linear component) in the second one. We shall comment on both later on.
 - [37] We refer the reader to [27] for a recent interesting application of c_{cs} with the velocity field as soft mode. The work in [27] relies on the fact that, in our language, the velocity has both linear and non-linear transformations.
 - [38] We have verified this without any of the approximations in Eq. (7) and including all the quadratic terms. The latter can in fact give also lower order contributions under the transformation in Eq. (8).
 - [39] We return here to the relation with the non-equal time Λ CDM correlators. Λ CDM dynamics does *not* break c_{cs} but the unequal-time correlators have a different behaviour in the squeezed limit w.r.t. their equal-time counterpart. This behaviour is similar to that of our reduced system in Eq. (1), originating the parallel of *Section II*.